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SUMMARY

Page

All manuscripts submitted for insertion in the "Telecommunication Journal" must reach the Editor at least 45 days before the date of publication.

Cover:

Static ESFI MOS chip for 4 kbit on 14.7 mm², produced by Siemens research laboratory. The 4.2 × 3.5 mm chip contains the actual storage matrix, the word circuit decoder (on the left), the bit circuit decoder (bottom) and the readout circuit (bottom left corner). The young lady behind the abacus doesn't appear to be entirely in the picture...

(Siemens)

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Some graphical considerations on Millington's method for calculating field strength over inhomogeneous earth

by
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Introduction

MILLINGTON's method for calculating field strength over inhomogeneous earth includes many calculation operations when there are several changes in earth constants along the path. These operations are not a problem when a computer is used, but they can give rise to errors when the calculations are done by hand. It is also easy to make mistakes when reading off so many values from the curves. A graphical method is easier to control, and it is then easier to see what happens along the path. A simplified approximate graphical method for Millington's method will be described here.

There will often be a need for a reliable and quick manual method to obtain the field strength at a certain distance. Furthermore there can be a need for finding the distance at which the field strength is of a certain value.

Eckersley's method

When a radio wave is transmitted over ground with earth constants σ and ε, the field strength at the receiver site is easily taken from the curves in Recommendation 368 of the International Radio Consultative Committee (CCIR). These curves are related to 1 kW effective radiated power (ERP), and the only extra operation to be done is to scale up the value on the curve with the number of decibels by which the actual ERP exceeds 1 kW.

When the radio wave is transmitted over ground with varying σ and ε, the field strength cannot be taken directly from the curves in Recommendation 368. To draw curves for all possible variations in earth constants σ and ε would be quite impracticable.

In order to use the existing curves a method was proposed by Eckersley. If the first part of the radio path is over ground with earth constants σ_1 and ε_1, the field strength at distance d_1 from the transmitter is found on the σ_1, ε_1 curve. On figure 1 this field strength is marked E_1.

Over distance d_2 with earth constants σ_2, ε_2, the curve for σ_2, ε_2 should be followed. But the field strength at the beginning of the second part, d_2, has to be the same as at the end of the first part, d_1. Therefore the σ_2, ε_2 curve for d_2 is displaced parallel up to the field strength at distance d_1, E_1, as shown on figure 1. The field strength at distance d_1+d_2 is then:

$$E_A = E_1 - (E_2 - E_3) \qquad (1)$$

If the first part of the path were over earth constants σ_2, ε_2 and the second part over σ_1, ε_1, we should first follow the σ_2, ε_2 curve and thereafter the parallel displaced σ_1, ε_1 curve.

Because of the vertically parallel displaced curves, Eckersley's method is not dependent of the scale used on the abscissa of the curves.

Eckersley's method does not give good agreement between calculated and measured values. And if we interchange receiver and transmitter, we will in most cases get quite different results. Eckersley's method therefore does not obey the principle of reciprocity.

Millington's method for one change in earth constants

Millington's method is designed to satisfy the principle of reciprocity by taking as the field strength at a certain distance the geometric mean between the values of Eckersley's method used both ways, as indicated on figure 2.

The geometric mean is the arithmetic mean when a linear log-scale is used. *When a linear decibel scale is used for the field strength, the geometric mean is half way between the two*

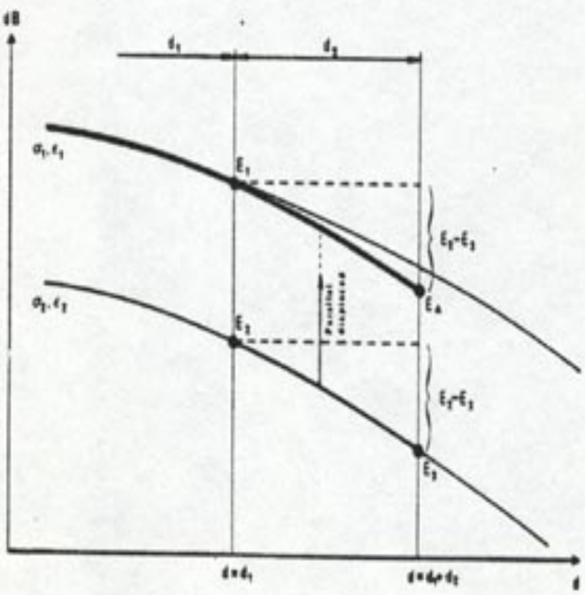

Figure 1
Eckersley's method

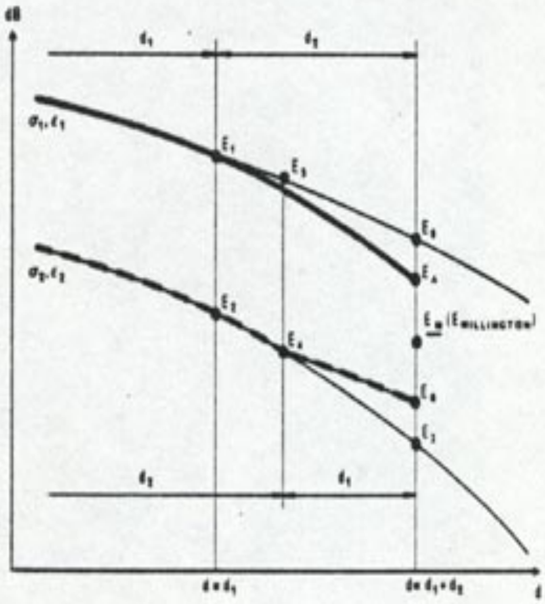

Figure 2
Millington's method for one change (to poorer earth constants)

values E_A *and* E_B *obtained from Eckersley's method used both ways:*

$$E_M = \frac{E_A + E_B}{2} \qquad (2)$$

Eckersley's method used both ways gives (see figure 2):

$$E_A = E_1 - (E_2 - E_3)$$
$$E_B = E_4 - (E_5 - E_6)$$

and the Millington field strength is then *(2)*:

$$E_M = \frac{E_A + E_B}{2} = \frac{E_1 - (E_2 - E_3) + E_4 - (E_5 - E_6)}{2} \qquad (3)$$

If d_2 varies from 0 and upwards, many points have to be calculated to find the resulting Millington curve. To make a quick graphical solution, it would be of great help if some special points or an asymptote could be found, so that the curve could be drawn without too much calculation.

An interesting point is when $d_2 = d_1$ $(d = 2d_1)$. Then $E_5 = E_1$ and $E_4 = E_2$ *(3)*:

$$E_M = \frac{E_1 - (E_2 - E_3) + E_2 - (E_1 - E_6)}{2}$$

$$E_M = \frac{E_3 + E_6}{2} \qquad (4)$$

that is, *when* $d_1 = d_2$, *the field strength is directly the geometric mean between the two curves at the distance* $d = d_1 + d_2 = 2d_1$.

The next step is to try to find an asymptote when the ordinate is labelled linearly in decibels. If $d_2 \rightarrow \infty$, then $E_4 \rightarrow E_3$ and $E_5 \rightarrow E_6$, and we get from *(3)*:

$$E_M = \frac{E_1 - (E_2' - E_3) + E_3 - (E_5 - E_5)}{2}$$

$$E_M = E_3 + \frac{E_1 - E_2}{2} = E_3 + m_1 \qquad (5)$$

which means that *the asymptote is parallel to the* σ_2, ε_2 *curve in* $+m_1$ *decibel distance.* m_1 is the half distance in decibels between E_1 and E_2 (see figure 2). When we then have the value E_2 at $d = d_1$, the geometric mean at $d = 2d_1$ and the asymptote, it is easy to draw the Millington field strength curve as indicated on figure 3.

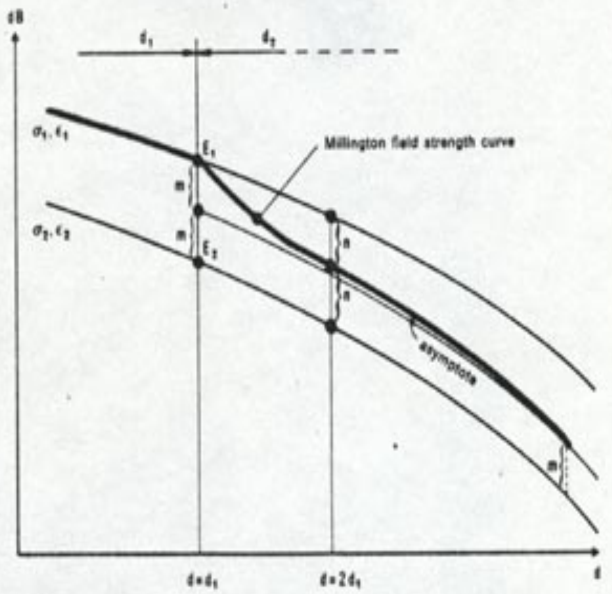

Figure 3
The Millington field strength curve for one change (to poorer earth constants)

Figure 3 shows Millington's method used when the first part of the path is over good earth constants and the second part is over poorer earth constants. If the first part is over poor earth constants and the second part over better earth constants, the result will be as shown on figure 4.

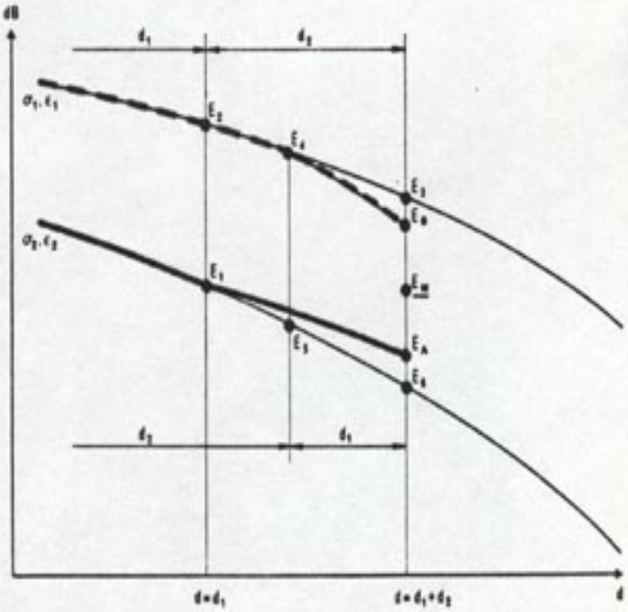

Figure 4
Millington's method for one change (to better earth constants)

If now d_2 varies from 0 and upwards, we will find that the Millington curve here also will pass through the point half way between the two σ, ε curves at the distance $d = 2d_1$. The asymptote is found to be parallel to the σ_1, ε_1 curve at distance $-m_1$ dB as indicated on figure 5.

Figure 5 shows one very interesting feature. When the first part of the path (d_1) is over poor earth constants σ_2, ε_2 and the second path is over good σ_1, ε_1 constants, the field strength can increase with distance from $d = d_1$. This is known as the *recovery effect*, and can be observed when the radio path goes from land to sea. Millington's method has been confirmed to be in good conformity with measured values.

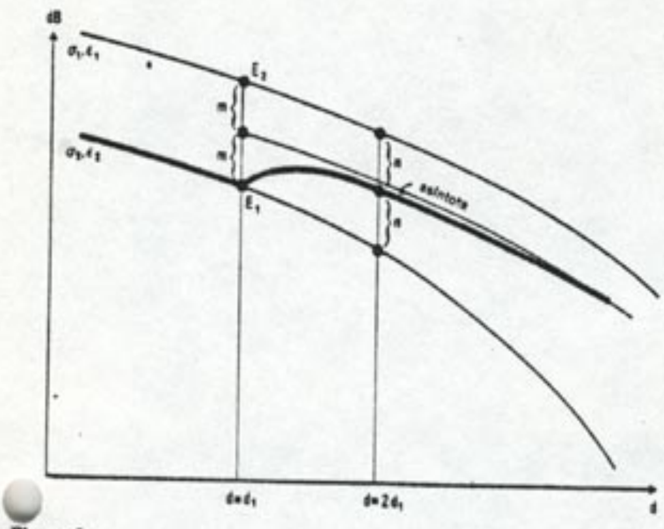

Figure 5
The Millington curve for one change (to better earth constants)

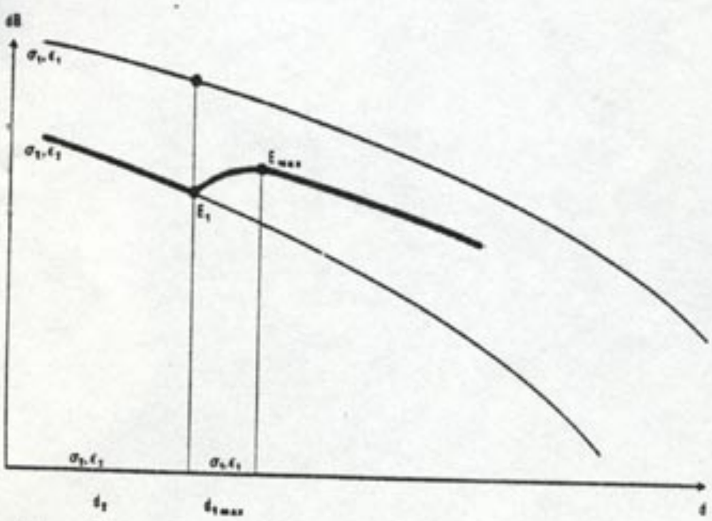

Figure 6
Gain when a distance with better earth constants is added to the path

Millington's method also gives a clear indication that it is important to have good electrical conditions in the ground to some distance from the transmitter, especially when we want to cover an area with bad σ and ε constants. The curve on figure 5 can be used the opposite way, as shown on figure 6.

If the transmitter is placed at a distance d_2 from a receiver and the earth constants are σ_2 and ε_2, a field strength E_1 is observed. And if the transmitter now is moved away from the receiver to an area with better earth constants, the received field strength can increase and reach a maximum. On figure 6 the maximum is reached at the distance $d = d_{1\max} + d_2$.

Milllington's method for more than one change in earth constants

On figure 7 Millington's method for two changes in earth constants is indicated. If we want to compare with calculation, we get from the same figure:

$$E_A = E_1 - (E_2 - E_3) - (E_4 - E_5)$$

$$E_B = E_6 - (E_7 - E_8) - (E_9 - E_{10})$$

$$E_M = \frac{E_1 - (E_2 - E_3) - (E_4 - E_5) + E_6 - (E_7 - E_8) - (E_9 - E_{10})}{2} \qquad (6)$$

To find a characteristic point on this curve is not so easy. But the asymptote can be found, and the result is very simple. The asymptote is reached when $d_3 = \infty$. Then $E_4 = E_5$, $E_7 = E_8$ and $E_9 = E_{10}$, and from *(6)* we get:

$$E_{m(d_3=\infty)} =$$

$$= \frac{E_1 - (E_2 - E_3) - (E_4 - E_5) + E_5 - (E_8 - E_8) - (E_{10} - E_{10})}{2}$$

$$E_{M(d_3=\infty)} = E_5 + \frac{E_1 - E_2}{2} + \frac{E_3 - E}{2} = E_5 + m_1 + m_2 \qquad (7)$$

The asymptote is then a curve parallel to the σ_3, ε_3 curve at distance $+(m_1 + m_2)$.

When the earth constants decrease along the path, the general expression is:

$$E_{Mn} = \frac{E_1 - (E_2 - E_3) \dots (E_{2n-2} - E_{2n-1})}{2}$$

$$\frac{+ E_{2n} - (E_{2n+1} - E_{2n+2}) - (E_{2n+3} - E_{2n+4})}{2}$$

and when

$$d_n = \infty, E_{2n} = E_{2n-1}, E_{2n+1} = E_{2n+2}, E_{2n+3} = E_{2n+4}$$

and so on, and the result is:

$$E_{(dn=\infty)} = \frac{E_1-(E_2-E_3)...(E_{2n-2}-E_{2n-1})+E_{2n-1}}{2}$$

$$E_{(dn=\infty)} = E_{2n-1}+\frac{E_1-E_2}{2}+\frac{E_3-E_4}{2}+...+\frac{E_{2n-3}-E_{2n-2}}{2}$$

or:

$$E_{(dn=\infty)} = E_{2n-1}+m_1+m_2+m_3+...+m_{n-1} \qquad (8)$$

that is, the asymptote is a curve parallel to the σ_n, ε_n curve at distance $+(m_1+m_2+m_3+...\ m_{n-1})$; m_n is here half the difference between the $\sigma_{n-1},\varepsilon_{n-1}$ and the σ_n, ε_n curves at the end of distance d_{n-1}.

This result is obtained when the σ, ε curves go to lower values along the path. If the σ, ε curves go to higher values along the path, the result is:

$$E_{(dn=\infty)} = E_{2n-1}-m_1-m_2-m_3-...-m_{n-1} \qquad (9)$$

where m_1 is half the difference between the σ_n, ε_n and the σ_{n-1}, ε_{n-1} curves at the end of d_1. m_2 is half the difference between the σ_{n-1}, ε_{n-1} and the σ_{n-2}, ε_{n-2} curves at the end of d_2, and so on.

If we compare these results with the Millington method for one change in σ, ε, it is obvious that we can evolve an approximate method which will give quick solutions. We first draw the Millington curve for *one* change at distance $d = d_1$, that is for the change between the σ_1, ε_1 and the σ_2, ε_2 curves. Thereafter the Millington curve for *one* change at distance $d = d_1+d_2$ between the σ_2, ε_2 and the σ_3, ε_3 curves is drawn, and this curve is displaced parallel to the field strength value at the end of d_1. The same procedure is repeated at the distance $d = d_1+d_2+d_3$ for the change between the σ_3, ε_3 and the σ_4, ε_4 curves, and so on. The resulting curve from this procedure will have the same sum of asymptote distances $m_1+m_2+m_3+...\ m_{n-1}$ as the asymptote distance for the exact curve. The approximate graphical method is explained on figure 8.

If we go to better and better transmission conditions, the graphical method is described on figure 9.

To have an approximation for asymptotes is not enough. It is therefore of interest to find the difference between the exact method and the approximate graphical method for *all abscissa values (all distances)*. The conditions for two changes in earth constants are first considered. From figure 10 the field strength at distance $d = d_1+d_2$ is (the graphical method is correct for *one* change):

$$E_X = E_{(d_1+d_2)} = \frac{E_1-(E_2-E_3)+E_a-(E_b-E_c)}{2} \qquad (10)$$

The Millington method for *one* change used at distance $d = d_1+d_2$ between curves for σ_2, ε_2 and σ_3, ε_3:

$$E_Y = \frac{E_3-(E_4-E_5)+E_6-(E_7-E_{11})}{2} \qquad (11)$$

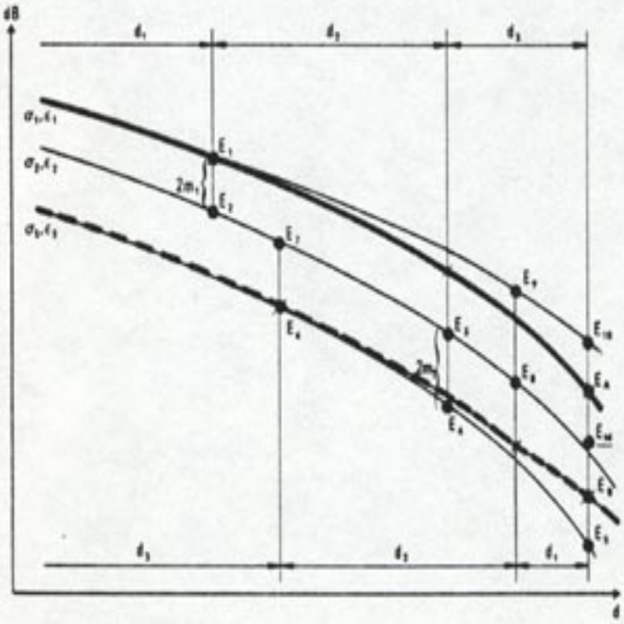

Figure 7
Millington's method for two changes (to poorer earth constants)

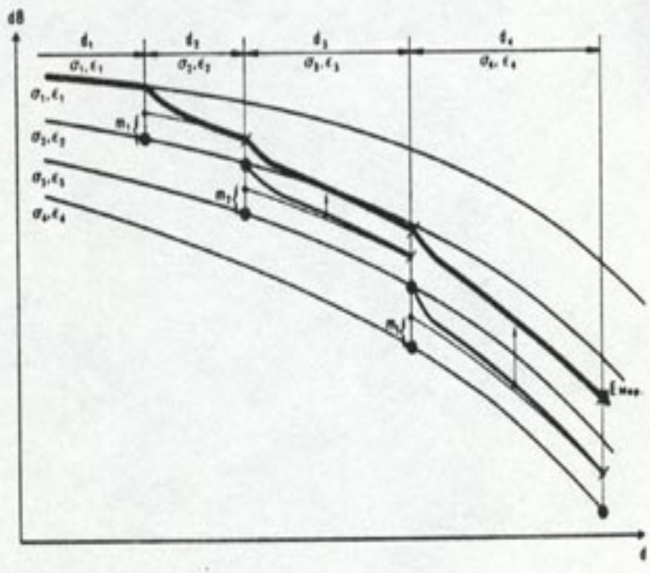

Figure 8
The simplified graphical method (for changes to poorer earth constants)

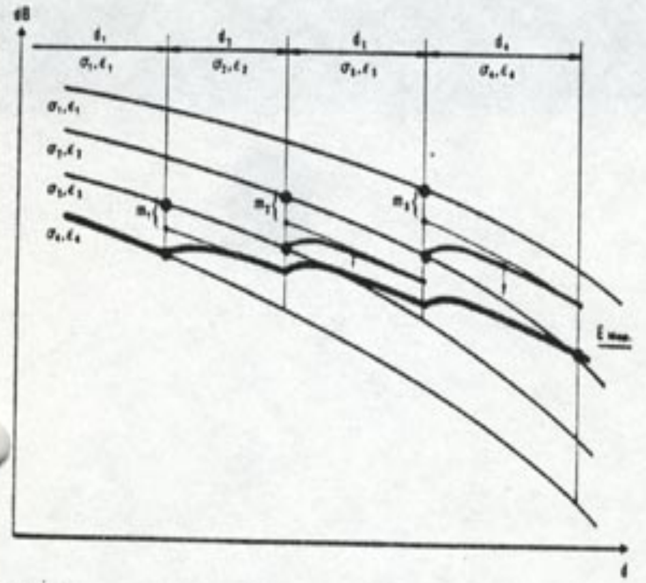

Figure 9
The simplified graphical method (for changes to better earth constants)

We must connect this value with the former value (E_x), and $E_x - E_3$ has to be added to E_y to get the approximate value at $d = d_1 + d_2$:

$$E_{Map} = \frac{E_3-(E_4-E_5)+E_6-(E_7-E_{11})}{2}$$

$$+\frac{E_1-(E_2-E_3)+E_a-(E_b-E_c)}{2} - E_3 \qquad (12)$$

The exact Millington field strength value is (see *(6)*):

$$E_M = \frac{E_1-(E_2-E_3)-(E_4-E_5)+E_6-(E_7-E_8)-(E_9-E_{10})}{2}$$

and the difference between the exact value and the approximate value is then:

$$E_M - E_{Map} = \frac{E_3+E_8-E_9+E_{10}-E_{11}-E_a+E_b-E_c}{2}$$

$$= \frac{(E_b-E_a)-(E_c-E_3)}{2} - \frac{(E_9-E_8)-(E_{10}-E_{11})}{2}$$

$$= \frac{\Delta 1}{2} - \frac{\Delta 2}{2} \qquad (13)$$

That is, *when the field strength curves for different earth constants are parallel, the graphical method is correct* (see figure 10). And if the curves are not quite parallel, the approximation is rather good because it is the half values of Δ which appear in the formula $E_M - E_{Map}$.

The formula $E_M - E_{Map}$ could now be developed for many changes in earth constants, and the result would be a number of small difference values. These differences can be both positive and negative, and can under some conditions cancel each other.

The variations in earth constants along the path can go in both negative and positive directions. It can be shown that also under such circumstances the approximation is acceptable. Only one example with σ_1, ε_1 for distance d_1, σ_2, ε_2 for distance d_2, and σ_3, ε_3 for distance d_3, will here be considered. These conditions are described in figure 11.

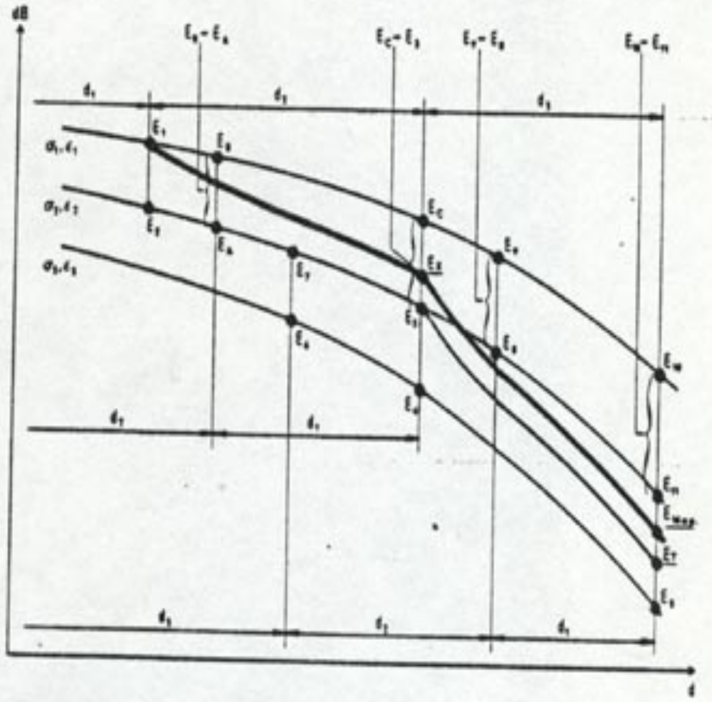

Figure 10
Figure to calculate the difference between the exact and the approximate methods

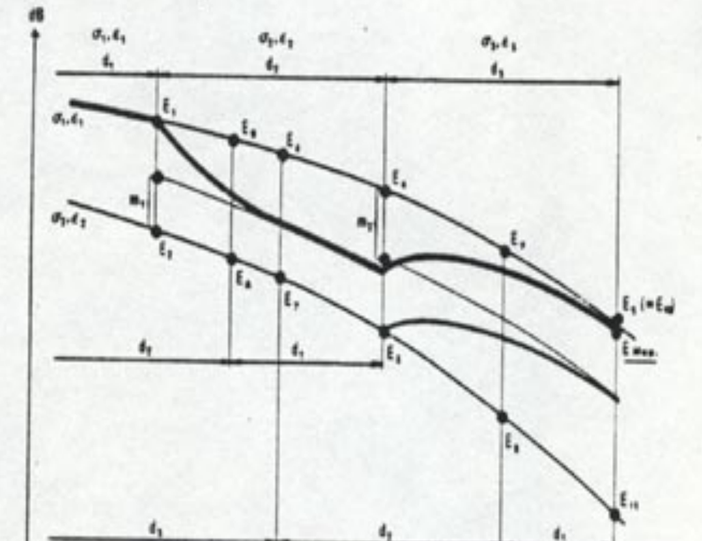

Figure 11
Variation both to poorer and better earth constants

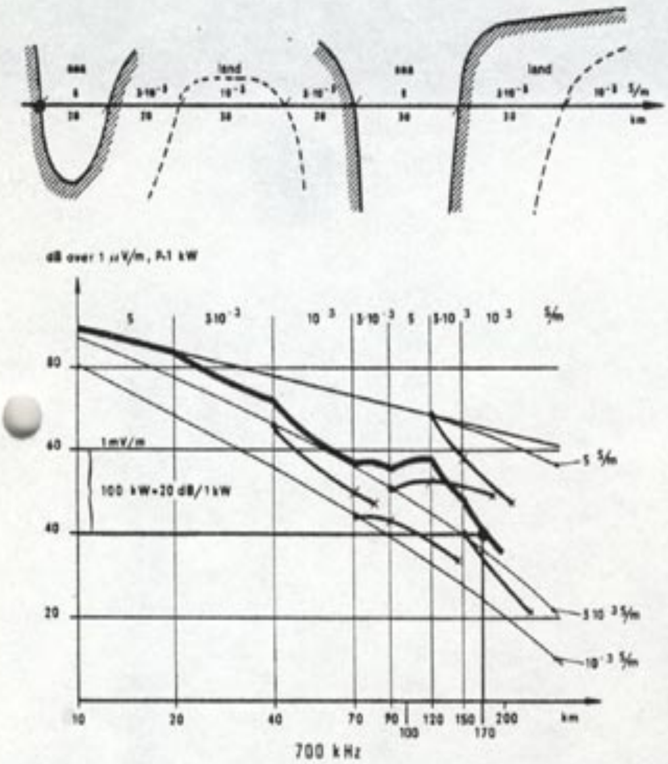

Figure 12
The approximate method used on several changes

The Millington method gives at the distance $d = d_1+d_2+d_3$ the field strength:

$$E_M = \frac{E_1-(E_2-E_3)-(E_4-E_5)+E_6-(E_7-E_8)-(E_9-E_5)}{2} \quad (14)$$

The Millington field strength at the distance $d = d_1+d_2$:

$$E_x = \frac{E_1-(E_2-E_3)+E_a-(E_b-E_4)}{2} \quad (15)$$

The Millington method used on the changes between the distance (d_1+d_2) *and* d_3:

$$E_y = \frac{E_3-(E_4-E_5)+E_6-(E_7-E_{11})}{2} \quad (16)$$

The field strength for the approximate method (graphical method) at distance $d = d_1+d_2+d_3$ is therefore:

$$E_{Map} = E_y+(E_x-E_3)$$

$$E_{Map} = \frac{E_3-(E_4-E_5)+E_6-(E_7-E_{11})}{2}$$

$$\frac{+E_1-(E_2-E_3)+E_a-(E_b-E_4)}{2} - E_3 \quad (17)$$

The difference between the exact and the approximate method *(14)-(17)*:

$$E_M - E_{Map} = \frac{E_3-E_4+E_5+E_8-E_9-E_{11}-E_a+E_b}{2}$$

$$= \frac{(E_b-E_a)-(E_4-E_3)}{2} - \frac{(E_9-E_8)-(E_5-E_{11})}{2}$$

$$= \frac{\Delta 1}{2} - \frac{\Delta 2}{2} \quad (18)$$

which is a result similar to that in *(13)*, that is, when the curves are parallel, the graphical method is correct. When they are not quite parallel, the differences will be so small that the result is acceptable.

This calculation could now be developed further, but always giving numbers of small differences. And it is found that the approximate graphical method will give good results for any variation in earth constants.

Conclusions

To make a quick manual calculation of ground wave field strength over inhomogeneous earth, an approximate graphical method can be used. This graphical method is based on the use of Millington's method for each section of the path, and the curves are displaced vertically so as to get a continuous curve.

The drawing of curves for each section can be done very quickly by using the point half way in decibels between the curves at $d = 2d_x$, where d_x is the distance for change of curves, and also by using the asymptote at the half way distance in decibels between the curves at the end of the former section.

Figure 12 describes how the graphical method can be used to find the distance where the field strength is 1 mV/m = 60 dB over 1 μV/m when the ERP is 100 kW = 20 dB over 1 kW.

The Millington method needs field strength curves for various earth constants σ and ε, but for one frequency at a time. Curves should therefore be made for earth constants at some frequencies. For manual calculations in the LF/MF bands field strength curves for earth constants at frequencies 150, 200, 300, 700 kHz, 1 and 1.5 MHz should be sufficient.

(Original language: English)